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Unified gauge theories and the gravitational cut-off

Q Shafi

Fakultät für Physik, Universität Freiburg, Hermann-Herder-Strasse 3, Freiburg, West Germany

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Abstract. Assuming that gravitation acts as a universal cut-off for strong, weak and electromagnetic interactions, we study some consequences for a class of unified gauge theories using the renormalization group method.

1. Introduction

The SU(2) × U(1) gauge model of Salam (1968) and Weinberg (1967) seems to provide a fairly good description of the current state of affairs in weak and electromagnetic interactions (for a recent review see Lee 1975). However, it is generally felt that this model is only one part of a more grand scheme chosen by nature, perhaps also encompassing the strong interactions.

Several such schemes have been put forward in the literature. One of their features is the prediction of the value of the Salam–Weinberg mixing angle, which is undetermined in the original model. However, renormalization effects can be expected to modify the mixing angle predictions (Georgi *et al* 1974). Clearly it is important to look for constraints (both theoretical and experimental) to limit the number of possible realistic models. This leads us to the main purpose of the paper. In this note we propose that gravitation be used to provide one such constraint†. That is, we assume that gravitation provides a universal cut-off for all other interactions. The effect of such a cut-off on unified gauge theories will be studied using the renormalization group (RG) (see Wilson 1971 for a discussion of electrodynamics in the presence of a cut-off). It will be seen that a fairly stringent constraint on the possible realistic models arises.

The plan of the paper is as follows. In § 2 we discuss quantum electrodynamics in the presence of a gravitational cut-off using the renormalization group. In § 3 we extend the discussion to unified gauge models which contain the SU(2) × U(1) model of Salam and Weinberg. In the calculation of the renormalization group parameters we assume that the gauge group of weak and electromagnetic interactions is SU(2) × U(1). We derive a constraint condition so that only models satisfying it need be considered. The renormalized value of the Salam–Weinberg mixing angle is calculable and can help in deciding between the various models. In § 4 we show how the constraint can be made more stringent when strong interactions are also taken into account. These are introduced via the colour $SU(3)_c$ so that the overall gauge group observed at present energies is assumed to be $SU(3)_c \times SU(2) \times U(1)$.

† The idea that gravitation provides a universal cut-off for the other interactions is an old one (see, e.g., the talk given by A Salam at Miami in 1971 and references therein).

2. Electrodynamics in the presence of a gravitational cut-off

To introduce the idea, consider electrodynamics in the presence of a gravitational cut-off $\Lambda \approx 1{\cdot}2 \times 10^{19}$ GeV. The momentum dependence of the effective electric coupling $e(\kappa)$, where κ denotes the momentum scale, is given by the RG expression (for $\kappa \gg m_e$, the electron mass)

$$\kappa \frac{d}{d\kappa} e(\kappa) = \beta(e(\kappa)). \tag{2.1}$$

For $|e(\kappa)|$ sufficiently small,

$$\beta(e(\kappa)) \approx b e^3(\kappa). \tag{2.2}$$

Equation (2.1) can then be integrated. To determine the integration constant we assume that $e(\kappa)$ remains sufficiently small up to $\kappa \sim \Lambda$, the gravitational cut-off, where it takes some value e_Λ. We obtain

$$e^2(\kappa) \approx \frac{e_\Lambda^2}{1 + 2be_\Lambda^2 \ln(\Lambda/\kappa)}. \tag{2.3}$$

Wilson (1971) considered the possibility where $e_\Lambda^2 \gg (2b \ln \Lambda/\kappa)^{-1}$ (without specifying a value for Λ so that this can always be arranged). Then

$$e^2(\kappa) \approx (2b \ln \Lambda/\kappa)^{-1}. \tag{2.4}$$

It is easy to see that (2.4) leads to difficulty for our case (where Λ is fixed). We have

$$\frac{e^2(\kappa)}{4\pi} \approx \frac{3\pi}{2 \ln(\Lambda/\kappa)} \tag{2.5}$$

where $b = (12\pi^2)^{-1}$ in lowest order of perturbation theory. Taking $\kappa = 1$ GeV, say, we obtain

$$e^2/4\pi \approx 0{\cdot}11, \tag{2.6}$$

a value much greater than the observed coupling constant $\alpha = (137)^{-1}$. Thus (2.4) is a bad approximation in our case and we keep equation (2.3). In fact we can estimate the value of e_Λ by using the known value of α. This gives

$$e_\Lambda^2/4\pi \approx (132)^{-1}. \tag{2.7}$$

Note that this value of e_Λ is not very significant since we have taken into account the contribution to β from only one type of charged particle. However, the essential point here is that the effective electric coupling increases as κ increases. This is, of course, a typical feature of an Abelian gauge theory (see also the discussion following (4.7)).

3. Extension to gauge models of weak and electromagnetic interactions

We turn next to the unified gauge models. The SU(2)×U(1) model of Salam and Weinberg is assumed to be embedded in some larger group G. The weak mixing angle is given by (Salam 1968, Weinberg 1967, Georgi *et al* 1974)

$$\sin^2\theta = \frac{g_1^2}{g_1^2 + C^2 g_2^2} \tag{3.1}$$

where g_2 and g_1 are the coupling constants associated with SU(2) and U(1) respectively and the constant C is defined by the relation

$$Q = I_3 - CY. \tag{3.2}$$

Now invariance under G implies that $g_1 = g_2$ so that

$$\sin^2\theta = (1+C^2)^{-1}. \tag{3.3}$$

However, since the gauge couplings are functions of the momentum scale, (3.3) is expected to get modified because of renormalization effects. In analogy to the Abelian case we can write down the following renormalization group equations for $g_1(\kappa)$ and $g_2(\kappa)$ (for $|g_1(\kappa)|$, $|g_2(\kappa)|$ sufficiently small and the boundary condition $g_1(\kappa), g_2(\kappa) \to g_\Lambda$ for $\kappa \sim \Lambda$, the gravitational cut-off):

$$g_1^2(\kappa) \approx \frac{g_\Lambda^2}{1+2b_1 g_\Lambda^2 \ln(\Lambda/\kappa)} \tag{3.4}$$

$$g_2^2(\kappa) \approx \frac{g_\Lambda^2}{1+2b_2 g_\Lambda^2 \ln(\Lambda/\kappa)}. \tag{3.5}$$

The effective electric coupling is given by

$$e^2(\kappa) = \frac{g_1^2(\kappa) g_2^2(\kappa)}{g_1^2(\kappa) + C^2 g_2^2(\kappa)}. \tag{3.6}$$

We note here that for κ of the order of present energies ($\kappa \sim 10$–10^2 GeV) and much smaller than all superheavy masses (these include, in particular, all the gauge bosons apart from the W and Z bosons of the Salam–Weinberg model), a theorem proved by Appelquist and Carrazone (1975) shows that b_2 and b_1 can be calculated in an effective field theory based on the gauge group SU(2)×U(1)†. This will be used shortly.

From (3.1), (3.4), (3.5) and (3.6) one can derive the relation

$$(1+C^2)\sin^2\theta = 1 - 2e^2(\kappa)(b_1 - b_2)C^2 \ln(\Lambda/\kappa) \tag{3.7}$$

which describes the effect renormalization has on the weak mixing angle θ. Note that for $\kappa \sim \Lambda$ we recover (3.3) as expected.

Next we assume, as in Georgi *et al* (1974), that the only multiplets of G that contribute differently to b_1 and b_2 are the gauge vector mesons themselves, which give respectively 0 and $-22\ (48\pi^2)^{-1}$ (see the discussion following (3.6)). Taking $\kappa = 10$ GeV and $e^2 \approx 4\pi\alpha$ in (3.7) leads to the following constraint on C^2 ($|\theta| \neq 0$ or $\pi/2$):

$$0 < C^2 < 2{\cdot}82. \tag{3.8}$$

Since C is determined by the underlying group, this is essentially a constraint on the unifying group G which embeds the SU(2)×U(1) model of Salam and Weinberg. Although in principle (3.8) still allows a large class of permissible unified models, the choice may be much more restricted when (3.7) is also taken into account. In order to see this we now discuss some specific models which have appeared in the literature (Weinberg 1972, Georgi and Glashow 1974, Fritsch and Minkowski 1975, Pati and Salam 1973, 1974, Elias and Swift 1975).

† The superheavy boson masses are of the order of Λ.

3.1. $SU(3)_L \times SU(3)_R$ model of Weinberg

For this case

$$C^2 = 3$$

which violates the constraint (3.8) and so gets ruled out as a suitable model for unifying the weak and electromagnetic interactions.

3.2. SU(5) model of Georgi and Glashow

Here $C^2 = 5/3$. Substitution in (3.7) gives

$$\sin^2\theta \approx 0{\cdot}15$$

which is to be compared with the group-theoretical value 3/8. This renormalization of θ should be taken into account when doing calculations. Experimentally one seems to find $\sin^2\theta \approx 0{\cdot}35 \pm 0{\cdot}12$†.

3.3. $SU_L(12) \times SU_R(12)$ model proposed by Fritsch and Minkowski

Here $C^2 = 11/5$ which then gives

$$\sin^2\theta \approx 0{\cdot}07$$

a rather low value in view of the above remarks.

3.4. Models based on the gauge groups $SU_L(2n) \times SU_R(2n) \times SU(4)$, for different values of n

When such groups are embedded in an appropriate universal group one obtains $C^2 = 4/3$, independent of n (Pati and Salam 1973, 1974, Elias and Swift 1975). This gives

$$\sin^2\theta \approx 0{\cdot}23$$

a value lying at the lower limit of the experimental numbers. Thus, from the point of view considered here, models belonging to this class are preferred.

We finally note that the above considerations remain essentially unchanged if we take $\kappa = 10^2$ GeV instead of 10 GeV. In particular the values of $\sin^2\theta$ considered above are unaffected to the two significant figures.

4. Inclusion of strong interactions

We now show how the constraint on C^2 (equation (3.8) above) can be made more stringent when strong interactions are also taken into account. We assume that the strong interactions are caused by the interaction of colour SU(3) quark triplets and an octet of massless colour gluons‡. When the momentum scale κ is large compared with all ordinary masses but small compared with all superheavy masses, the κ dependence

† See, e.g., the Proceedings of the 1974 London Conference and also Lee (1976).

‡ See, e.g., the Oppenheimer Lecture given by M Gell-Mann in 1974 at Princeton.

of the quark–gluon coupling constant g_3 is governed by a renormalization group equation analogous to (3.4) and (3.5):

$$g_3^2(\kappa) \approx \frac{g_\Lambda^2}{1 + 2b_3 g_\Lambda^2 \ln(\Lambda/\kappa)} \tag{4.1}$$

where b_3 is calculated in the field theory based on the gauge group SU(3) (Appelquist and Carrazone 1975).

Next, let us assume that the fermion contribution to b_3 in (4.1) arises from four triplets of coloured quarks (including a triplet of charmed quarks). One them obtains (Politzer 1974a, b)

$$b_3 = -25(48\pi^2)^{-1}. \tag{4.2}$$

From (3.4), (3.5) and (3.6) and the remarks following (3.7), one finds

$$(1 + C^2)(g_\Lambda^2)^{-1} \approx (13{\cdot}35 - 1{\cdot}4C^2). \tag{4.3}$$

Note that this result is representation dependent to the extent that we assume that there are eight SU(2) doublets (six quark doublets and e–ν_e, μ–ν_μ doublets). Substitution of (4.2) and (4.3) in (4.1) leads to the following constraint on C^2 ($\kappa = 10$ GeV):

$$C^2 < 1{\cdot}54. \tag{4.4}$$

We see that this constraint is more stringent than the one obtained before (equation (3.8)) by considering only the weak and electromagnetic interactions. Moreover, provided that the corrections to (4.4) from higher-order contributions to the β functions are sufficiently small, the model of § 3.2 is ruled out as a suitable unifying group.

From (4.1), given C^2, $g_3^2(\kappa)$ can be calculated. Taking $C^2 = 4/3$ (the model of § 3.4) and $\kappa = 10$ GeV gives

$$g_3^2/4\pi \approx 0{\cdot}28. \tag{4.5}$$

This value of $g_3^2/4\pi$ is in accord with current ideas on the quark–gluon theory of strong interactions (Politzer 1974a, b). Similar estimates can also be made for g_1^2 and g_2^2. One obtains

$$g_1^2/4\pi \approx 1/80 \tag{4.6}$$

$$g_2^2/4\pi \approx 1/31. \tag{4.7}$$

Equations (4.5), (4.6) and (4.7) show clearly the disparity in strengths of the various couplings (at present energies) which arises due to renormalization effects.

An interesting consequence of the constraint (4.4) is that the effective electric coupling decreases slightly as κ increases. The actual decrease is model dependent. Thus, for the model of § 3.4, the percentage decrease is approximately 0·25% as κ increases by an order of magnitude. An experimental observation of such an effect would provide a strong argument in favour of embedding electromagnetism in a non-Abelian gauge theory. We stress, however, that the non-Abelian nature of the theory was not enough to lead to this conclusion. The constraint on C^2, provided by (4.4), plays an essential role.

5. Discussion

The conjecture that gravitation provides a universal cut-off for strong, weak and electromagnetic interactions is very appealing. We have shown that, when used in conjunction with the renormalization group method, it enables one to evaluate renormalization effects in gauge theories. Thus, one can see explicitly why strong interactions are strong in unified theories of strong, weak and electromagnetic interactions. Moreover, it imposes a constraint on any realistic unified model. For models satisfying this constraint one predicts a decrease of the effective electric coupling as the momentum scale increases. The renormalized value of the Salam–Weinberg mixing angle is also calculable and can be used, along with other model-dependent quantities, to single out the physically relevant model. Finally, we remark that according to the ideas discussed here, unified models of strong and non-strong interactions contain superheavy bosons with masses of the order of the gravitational cut-off†. This follows since the unification of the forces is expected to set in when symmetry breaking effects can be neglected.

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† Note that the superheavy boson masses cannot exceed the gravitational cut-off. This is to be contrasted with the approach of Georgi *et al* (1974) where no such constraint applies and superheavy bosons with masses well beyond the gravitational cut-off are allowed.